The Mechanics of Friction in Rope Rescue

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Frictional forces play an important role in rope rescue. Friction force helps control the lowering of rescuers, however, friction force fights against the rescuer during a raise. Since friction in rope rescue can change exponentially with the rope geometry and the coefficient of friction, understanding the factors that affect rope friction is essential in technical rescue.

By applying a simple friction law derived for a capstan to friction forces in a break tube, rappel rack, and a figure-eight, we gain a better understanding of the behavior of these devices. While the conclusions drawn from this study are not counter to the current beliefs and practices within the rescue community, this study quantifies why some friction devices perform better than others.

In addition, these same friction laws can be used to better understand the frictional force for a rope going over a rock face. For example, the interaction of static and dynamic coefficients of friction can explain the bouncy ride that rescuers sometimes feel when they are at the end of a long haul system. Rope dynamics generated by friction can be estimated given the amount of rope, the weight of the load, the rope modulus, and the frictional force.

Friction Law

Tangential forces generated between contacting surfaces are known as frictional forces. These tangential forces resist motion up to a point. Experiments have shown how the limiting tangential force that can resist motion is proportional to the normal force along the contact surface. Thus, for impending motion the frictional force, F_f is proportional to the normal force, F_N

$$F_f = \mu F_N \tag{1}$$

where μ is called the ***coefficient of friction***.

Once the maximum frictional force is exceeded, then sliding will occur. There will still be resistance to slippage, and the magnitude of the tangential force for a sliding surface will also be proportional to the normal force. However, the frictional force for sliding contact will be lower.

The ratio of the limiting frictional force to the normal force for no slipping is called the static coefficient of friction. For sliding surfaces, the term dynamic coefficient of friction is used. The transition between static an sliding coefficient of friction is shown in Figure 1.

Automobile drivers know when the tires are sliding a car will have less breaking power than if the tires are not locked. Anti-lock breaks take advantage of the fact that sliding friction is less than static friction.

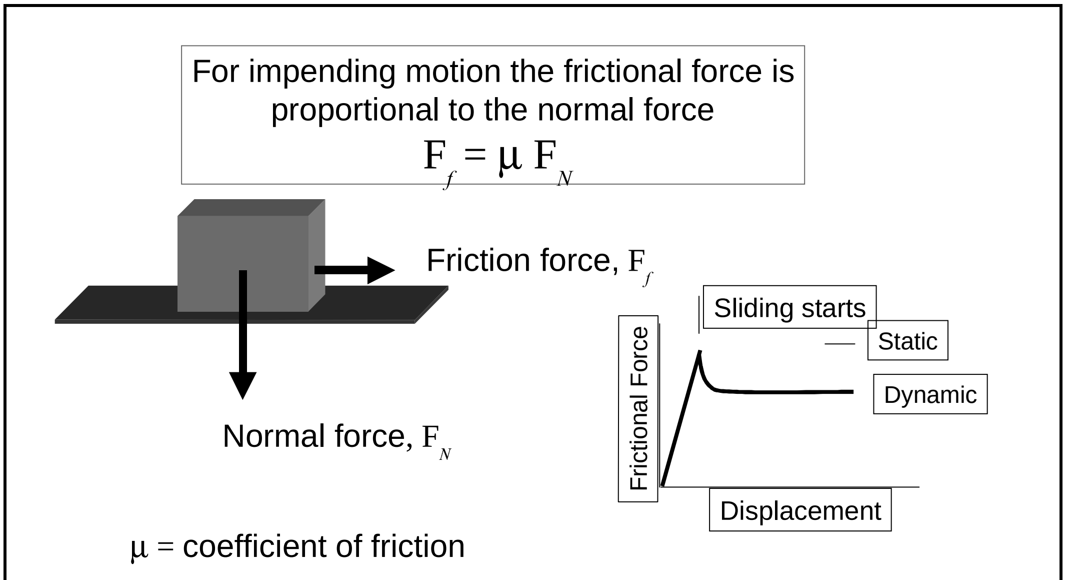

Figure 1: Friction Law.

Figure 2 shows a friction example for a brick that must be slid across a floor. We all know from real life experience that even though the two different block orientations will have a much different contact areas, the amount of force required to start the block sliding will be the same for both orientations. If the forces were different, then construction workers would always try to stack material so that they would be easy to slide.

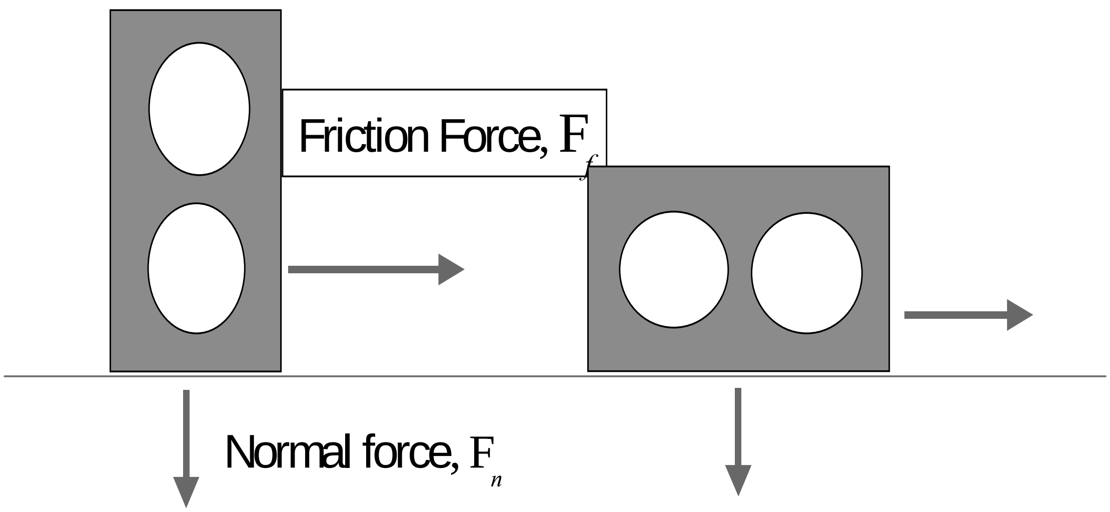

Figure 2: Friction Example.

The force required to slide the blocks depends only on the weight of the block and the coefficient of friction between the block and the sliding surface. In order to see how the contact area cancels from the friction equation, consider the normal stress defined as:

$$\sigma_n = F_n/A \tag{2}$$

The friction stress will be $\mu\sigma_n$ and the frictional force will be $F_f = \mu\sigma_n A$ to give:

$$\frac{F_f}{F_n} = \frac{\mu\sigma_n A}{\sigma_n A} = \mu \tag{3}$$

When the frictional forces are computed using the assumption that the limiting tangential stress is proportional to the normal stress, the areas council to give the ratio of the tangent force to the normal force independent of the contact area.

To a good first approximation, the independence of the friction law to contact area also applies to ropes. If the load is extreme, then the above equations may not be accurate. However, the friction law presented here should provide a very good approximation for most loads.

The Capstan Friction Equation

In Figure 3, the differential forces for tension of a rope over a drum are shown assuming impending slipping and no bending strength. Figure 4 shows the finial equation which is known as the capstan friction equation. Note that in order to understand the rest of this paper, the reader need not understand the derivation of these equations. The derivation is presented for completeness and can be found in J. L. Meriman.

As a rope bends over a small segment of a drum, the tension in a rope will increase from T to T+dT in an angle $d\theta$. The normal force is the differential dN, since it acts on a differential of area. The frictional force is μdN, and acts to oppose slippage.

Equilibrium in the x direction requires the sum of forces in the x direction equal to be zero,

$$\sum F_x = 0 \tag{4}$$

$$T\cos\frac{d\theta}{2} + \mu(dN) - (T + dT)\cos\frac{d\theta}{2} = 0 \tag{5}$$

which reduces to

$$\mu dN = dT \tag{6}$$

if one recalls that cosine of a differential is unity and the product of two differentials can be neglected. Equilibrium in the y direction, similarly, gives

$$dN - (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2} = 0\,, \tag{7}$$

which reduces to

$$dN = Td\theta. \tag{8}$$

The normal force can be eliminated from equation 6 and 8 to give a differential equation for T in terms of the contact angle θ.

$$\frac{dT}{T} = \mu d\theta\,.$$

Integration over the total contact angle gives the ratio of the tension force in terms of the coefficient of friction, μ, and the contact angle β.

$$\int_{T'_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} u\,d\theta \tag{9}$$

or after integration of equation 9 we get,

$$\ln\frac{T_2}{T_1} = \mu\beta, \tag{10}$$

which reduces to the capstan equation:

$$T_2 = T_1 e^{\mu\beta}. \tag{11}$$

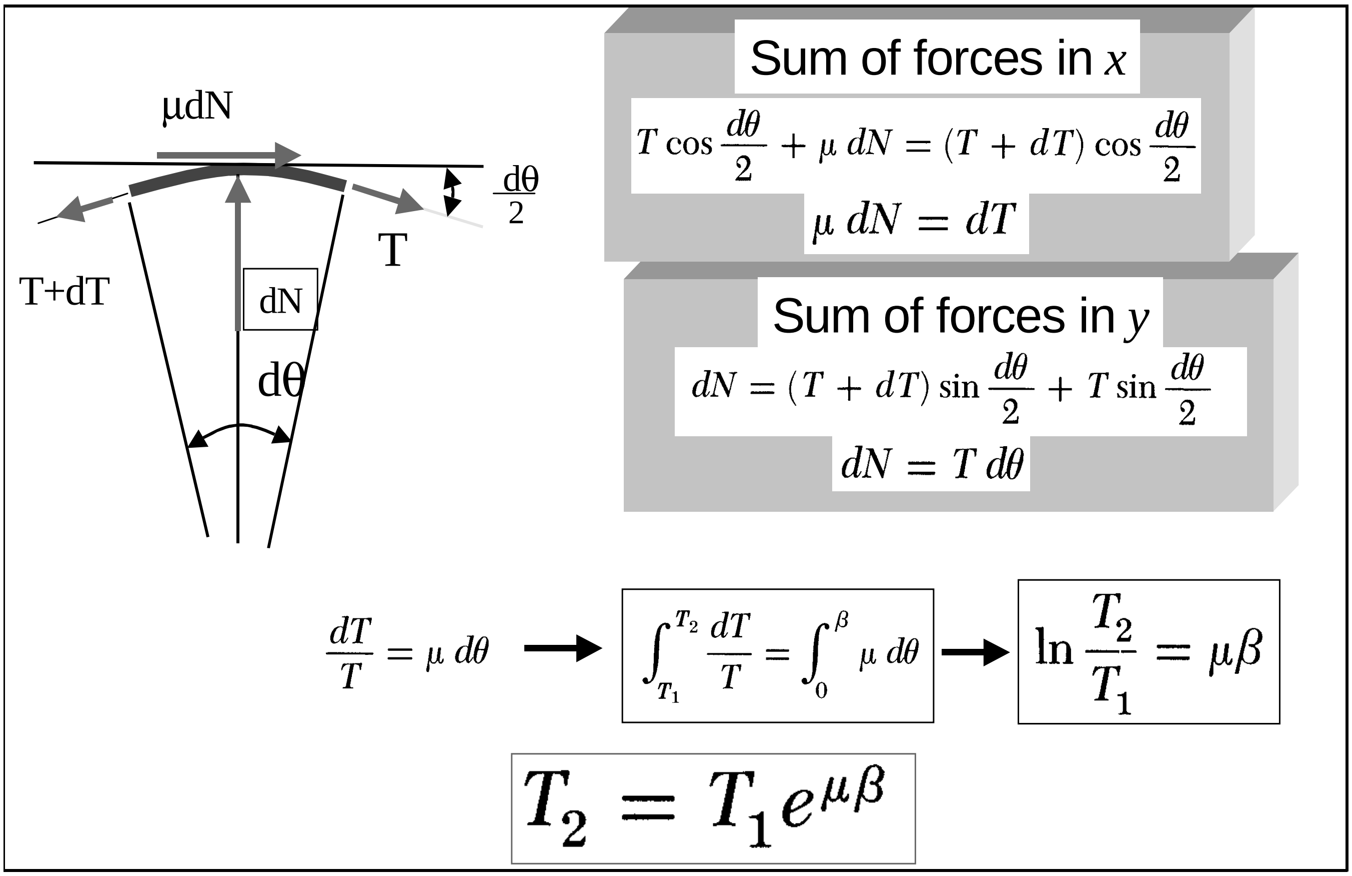

Figure 3: Amontons’ friction law for a flexible belt. (J. L. Meriam)

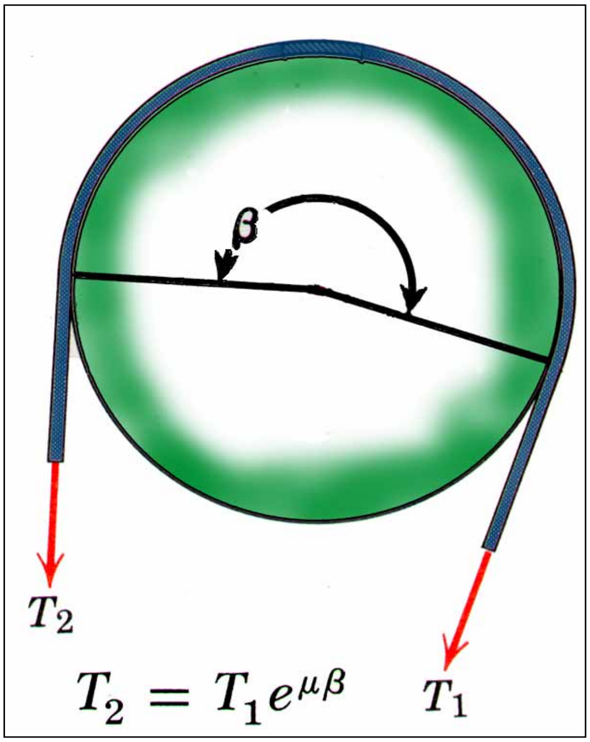

Figure 4: The capstan equation for belt friction.

Figure 4 summarizes the capstan equation for friction over a drum. Using this simple friction law leads us to conclude that the frictional forces for a rope depends only on three things:

- the tension in the rope
- the coefficient of friction
- the total angle of contact

For the friction model we have considered, the friction will increase exponentially with the coefficient of friction and the contact angle. Just like the sliding contact block, the solution is independent of the contact area, and thus independent of the radius of bend and the size of the rope.

The friction on a rope can vary greatly depending on the rope conditions. If the rope is muddy or wet, then the friction will be reduced. If the rope is old or the outer sheath is worn, then the friction may increase. For the examples shown here, we will assume that the rope is uniform and that the friction is constant over the length of the rope. The real world will be different; however, we will still be able to gain understanding of the real world by studying some "ideal" friction cases.

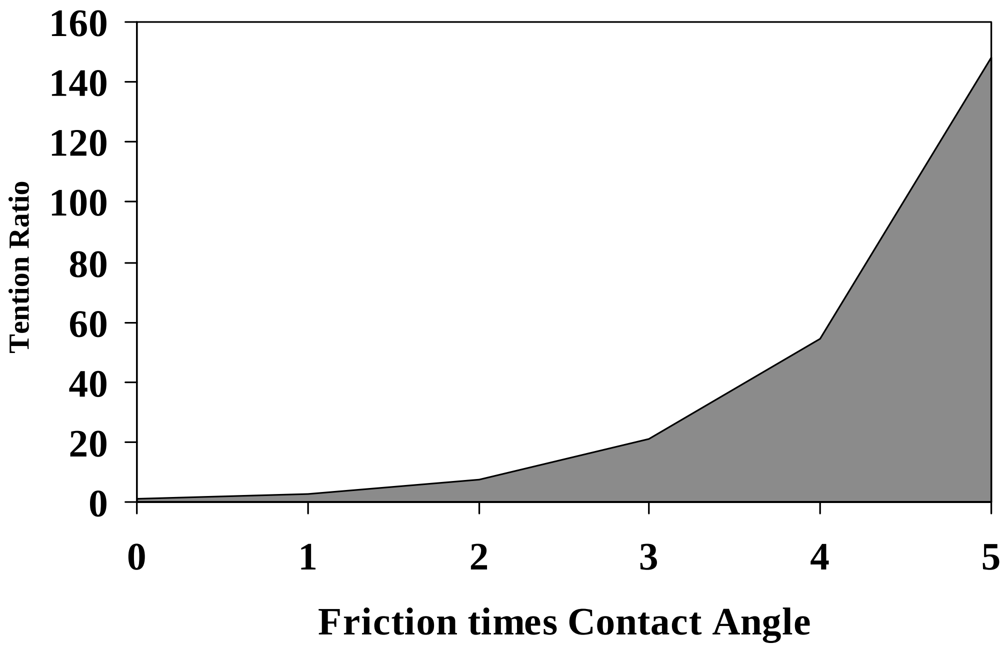

Figure 5: Example of exponential function of contact behavior.

The plot in Figure 5 illustrates the dramatic increase of an exponential function. For this graph, the x axis represents the coefficient of friction times the contact angle. The Y axis gives the ratio of the tension force T_1/T_2. Note that for a contact angle of zero or a friction coefficient of zero, the ratio is 1.0. That is expected. As the product of the coefficient of friction and the contact angle increase, the function increases slowly at first. Then, as with any exponential function, the value grows rapidly.

For the capstan equation, the contact angle must be in radians. Recall that 360 degrees or one revolution is equal to 2π radians.

Capstan Equation applied to breaking devices

In this section, I apply the capstan to some of the common breaking devices used in rope rescue. For comparing these different devices, I assumed the same coefficient of friction for each device. I assumed (did not measure) the coefficient of friction to be 0.25. The coefficient of friction for rope on aluminum can vary according to the rope type and condition. Mud, water, ice, and oil can all affect the coefficient of friction. I used $\mu = 0.25$ as an average number for comparison.

The Tube

The break tube is designed for lowering rescue loads. The idea for the tube was borrowed from the sailor's capstan. As can be seen in the calculations shown in Figure 6, the number of wraps will dramatically increase the frictional force.

For one wrap, the estimated ratio of holding force to load would be 10-to-1. For two wraps, the angle of turn increases to $\beta = 5\pi$ or 900 degrees. This will generate a 50-to-1 load holding capability. The effects of the exponential can really be seen when three wraps are used. For this case, the holding ratio jumps to 250 to 1.

For a 600-pound rescue load, one would expect to use 60 pounds of holding power if only one wrap is used. For two wraps, only 12 pounds would be required. If the three wraps are used, then only two to three pounds of holding force are required. The nice thing about the tube is that the large bending radius makes it easy to feed the rope, which means that three wraps are very easy to use.

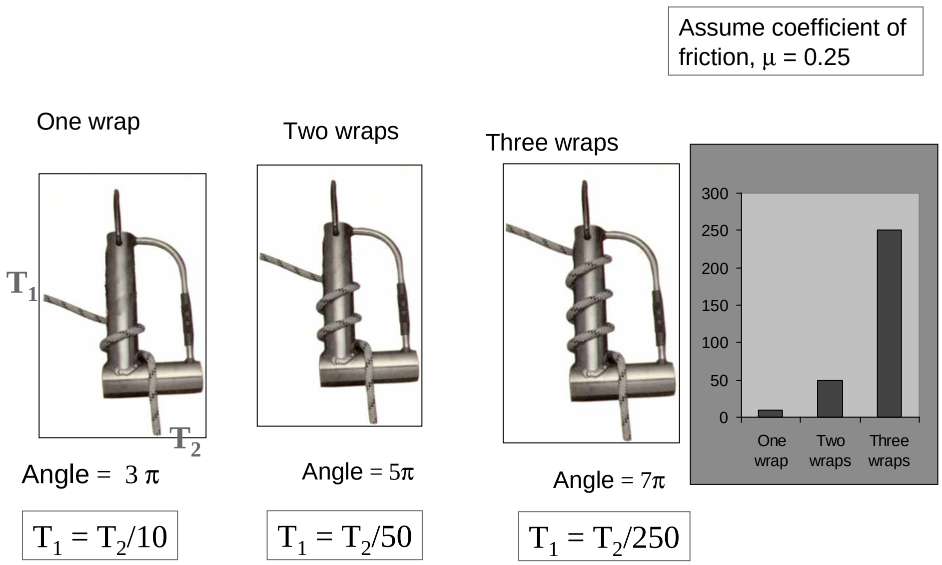

Figure 6: An analysis of the break tube for different frictional geometries.

The Rappel Rack

Figure 7 shows different geometries for the rappel rack. As can be seen in this illustration, one advantage of the rappel rack is the ability to increase the friction by changing the geometry. Simply sliding the bars changes the contact angle. For the average bar position (center of figure), I estimated the total contact angle of a rack to be $\beta = 560^o$ or 3.2 π radians. This total contact angle would give an average breaking force of 12-to-1 for a coefficient of friction of 0.25. The maximum angle change should be about 800 degrees, which gives a breaking force ratio of 31-to-1. The minimum angle change with only 5 bars fully spaced is about 4-to-1. This configuration could be very useful for a particularly heavy rope. Note that if only 4 bars are used, then the breaking force could drop to a dangerously low level.

For a 600-pound rescue load, the maximum holding geometry would require a 30-pound breaking tension. For a load of 30 pounds below a rappeller, the minimum friction geometry would allow a 120-pound weight to slide down the rope.

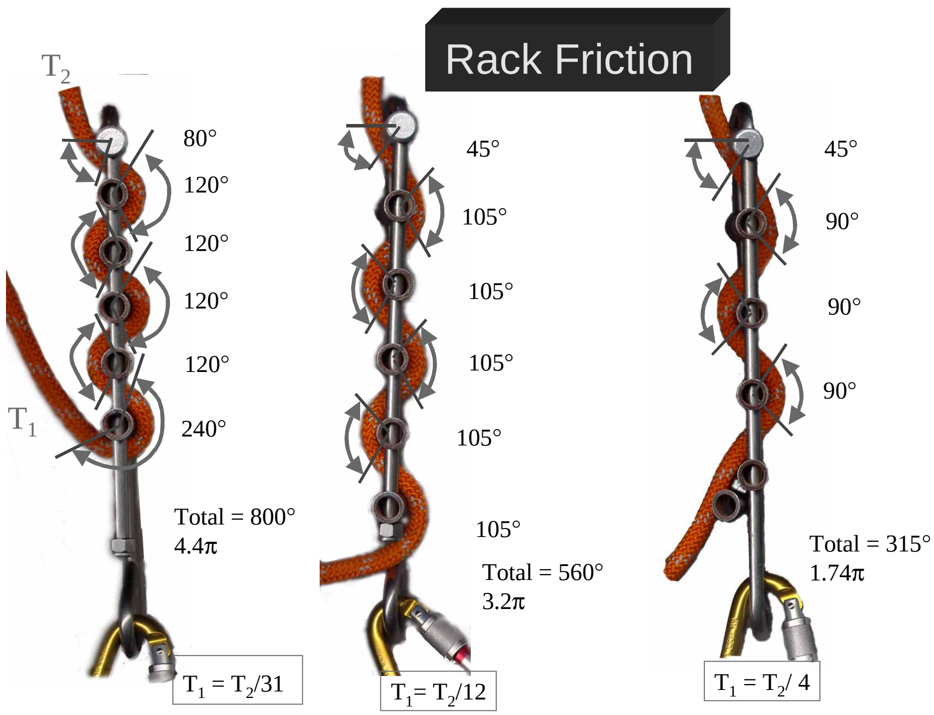

Figure 7: An analysis of break rack friction for different frictional geometries.

The Figure-Eight

The geometry of the figure-eight breaking device makes it difficult to estimate the contact angle. My estimate was 540 to 630 degrees for the configuration shown in Figure 8. This should give a holding ratio of 10-to-1 to about 15-to-1, which is comparable to the mid-range contact angles for a rappel rack. For a 600-pound load, a 40- to 60-pound holding force will be required.

For a heavy rope, the holding ratio cannot be reduced. For a 30-pound load hanging below the rappeller, the rappel weight would weigh about 450 pounds. The only solution is to “pull” oneself down the rope.

Which breaking device is best?

The break tube with three wraps has by far the greatest ratio of friction to load. The large bends in the tube also make it easy to control. Even though a tube with three wraps can generate a holding ratio of 250, it is still very easy to feed the rope into the tube.

The advantage of the rack is that moving the break bars can change the contact angle. For rappels where the rope has considerable tension, the ability to adjust the friction can be an advantage.

Figure-eight descending devices cannot be adjusted under tension, tend to twist the rope, and can be difficult to control. Based on these calculations, one can conclude that the figure-eight does not have a large range of frictional adjustment when compared to the break rack or the break tube.

While no conclusions should be based on calculations alone, these simple idealized examples show the effects of the exponential relation between the friction contact angle and the minimum/maximum breaking power of these devices.

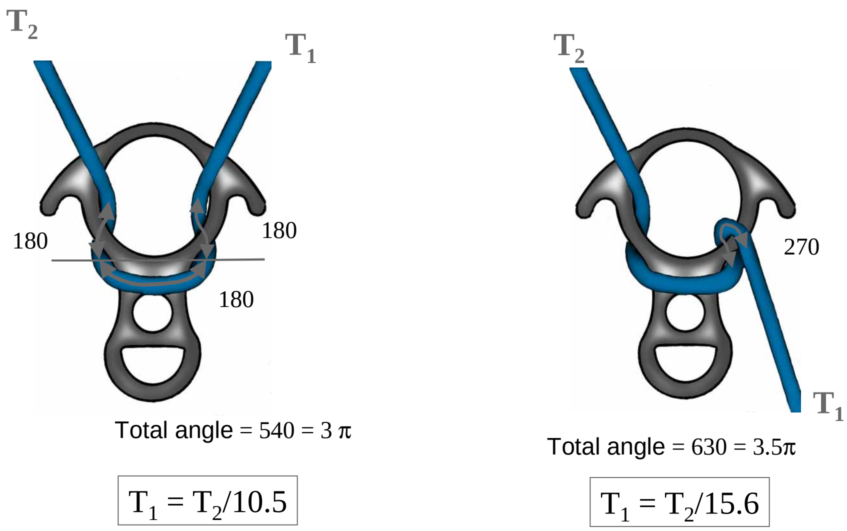

Figure 8: An analysis of the figure-eight breaking device for different frictional geometries.

Rope Edge Friction

Next we turn our attention to the mechanics of friction as a rope goes over the edge of a cliff or rock face. As you might expect, the angle that a rope turns as it goes over the edge can greatly effect the frictional force. What is unexpected is that the contact area does not affect the frictional forces.

The example shown in Figure 9 is intended to illustrate the frictional forces generated as a rope contacts a cliff face. For this example, the coefficient of friction was assumed to be 0.4. First a 45-degree change in angle was considered. The increased/decrease in tension due to friction was only 1.36. For a more typical 90-degree turn, a 1.87 change in tension occurred. The effect of contact angle really stands out when for a 135-degree change in angle. This geometry results in 2.60 times more tension. While this high ratio may be great for lowering, the resulting tension for a rescue load of 600 pounds would be almost 1,560 pounds. Remember that a single prusik will usually slip below this load.

The most common rescue situation that could generate a greater than 90 degree bend would be a ridge that has an anchor located downhill from the top of the ridge line. **Bottom line: avoid large changes in rope angle.**

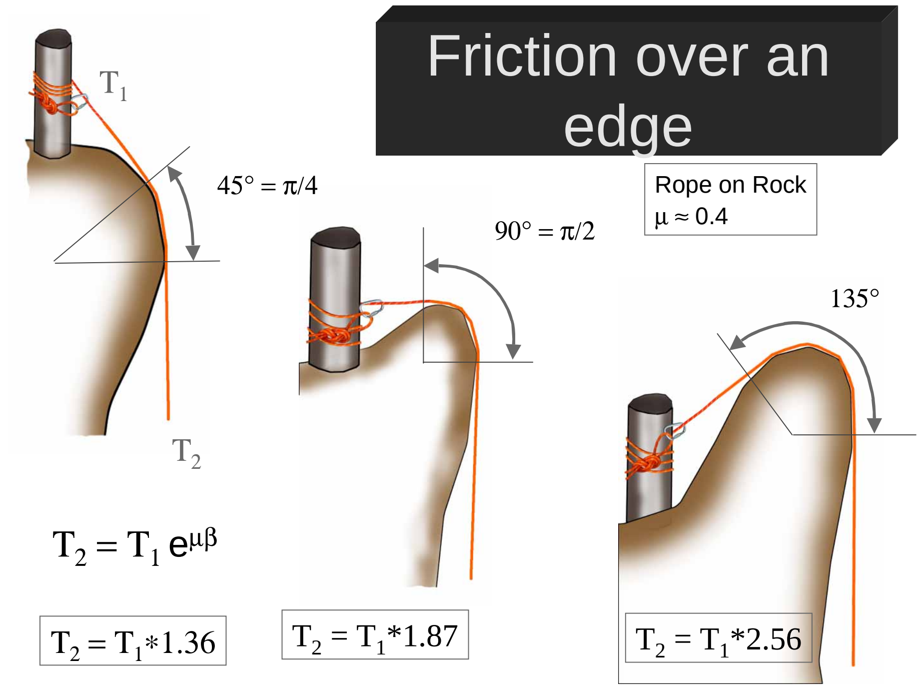

Figure 9: An analysis of friction for a rope that goes over an edge.

Typical Z-System Forces

The next example is intended to illustrate the effect high frictional forces can have on a typical haul system. The geometry for a typical Z-system raise is shown in Figure 10. A 600-pound rescue load was used in the computations. The resulting rope tension generated by two contact points with 45-degree bends was computed assuming a range of coefficients of friction. Figure 11 shows the resulting tensions for each rope segment plotted relative to each other.

For a coefficient of friction of $\mu = 0.45$, the maximum force in the rope as almost 1,500 pounds. This high frictional force almost cancels the mechanical advantage gained by the 3:1 Z-system.

If we assume that the edges have been protected, then the friction will drop. Assuming that the friction coefficient drops to 0.25, then the maximum load would be less than 900 pounds, which is still high, but nowhere near the 1,500-pound load. The required haul force would still be 300 pounds, which results in only a 2:1 haul system instead of the ideal 3:1. (I did not account for the loss in the pulleys.)

The example shown in Figure 10 also illustrates that the frictional force depends only on the total change in angle.

$$T_3 = T_2 e^{\mu\beta_{23}} = T_1 e^{\mu\beta_{12}} e^{\mu\beta_{23}} \qquad (12)$$

$$T_3 = T_1 e^{\mu(\beta_{12} + \beta_{23})} \qquad (13)$$

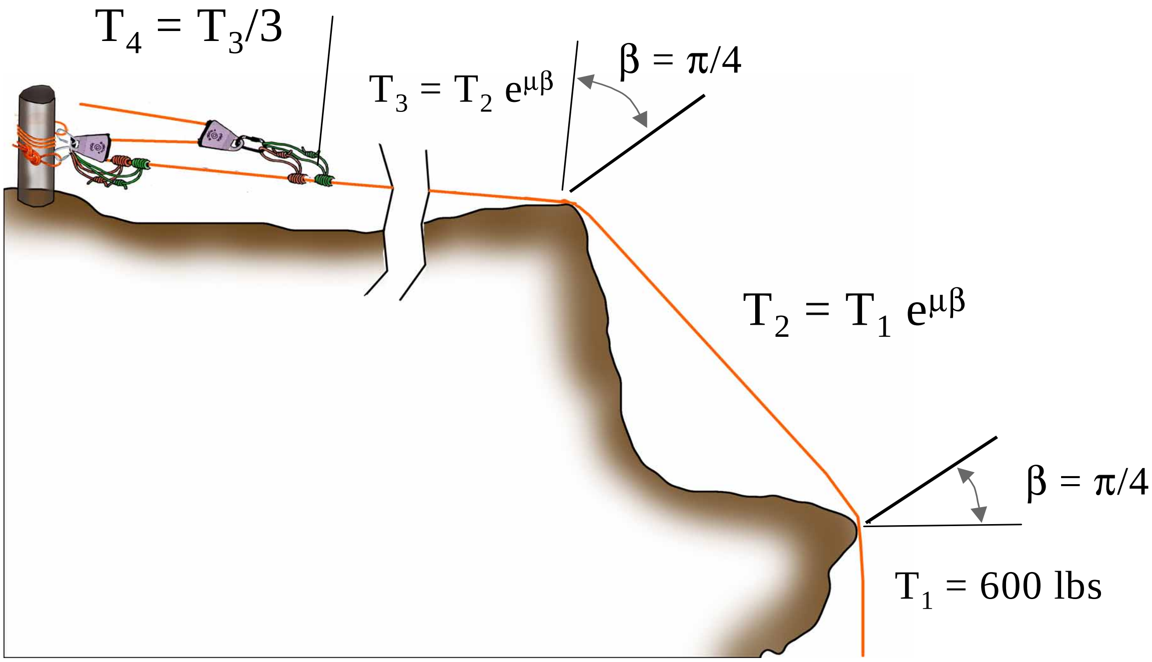

Figure 10: Rope tensile loads for a typical haul system.

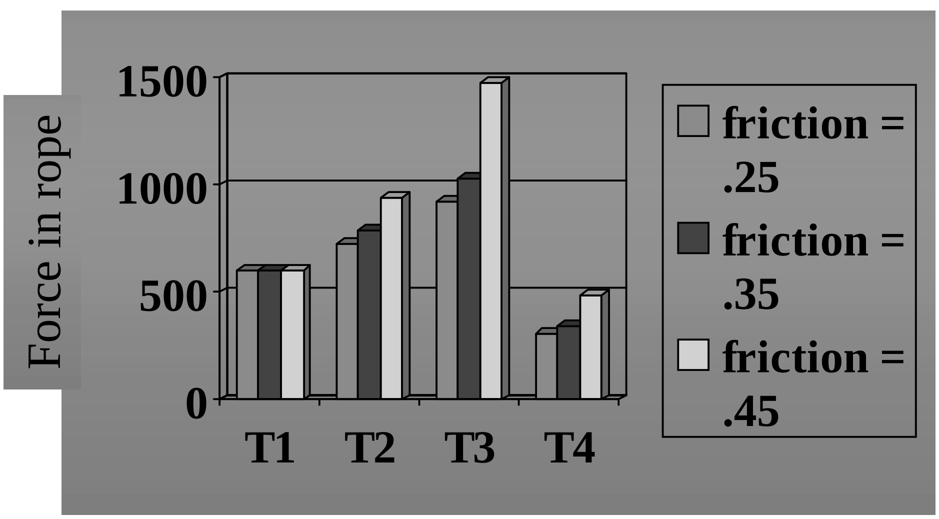

Figure 11: The force in rope segments T1 though T4 for the geometry shown in Figure 10.

Rope Stretch and Bounce due to Friction

Rope stretch can interact with friction in strange and sometimes dangerous ways. In this section, the forces and strain energy stored in a rope will be computed and used to predict the amount of bounce in a rope system.

Rope Stretch

Ropes can be viewed as a spring because they stretch as they are loaded. The amount of stretch depends on what is called the spring rate or stiffness. Rope stretch is a function of the rope size and construction. In general, a larger diameter rope will have less stretch under a given load than a smaller diameter rope. Figure 12 shows a plot of rope stretch for different ropes. From this plot, we see that static ropes all fail at about the same percent stretch. The higher loads for the larger diameters are the results of more nylon fibers. The twisted construction of the dynamic rope allows it to stretch more before it fails.

The ***stiffness*** of a rope can be used to compute the tension based on the change in length

$$T = K*d \qquad (14)$$

The stiffness depends on the length of rope and the rope modulus. The ***rope modulus*** is the slope of the load deflection curve for a unit length of rope. The stiffness is defined as

$$K = M/L. \qquad (15)$$

A long rope will not be as stiff as a short rope. PMI 11 mm static has a modulus of about 19555 lbs. As an example, a 200 ft rope would have a stiffness of

$$K = 19555\text{lbs} /200 \text{ ft} \quad = 97.7 \text{ lbs/ft} \qquad (16)$$

A 200 ft rope supporting 1000 lbs would stretch:

$$T = K*d \qquad (17)$$

or

$$d = T/K \qquad (18)$$

to give E = 1000/97.7 = 10.2 ft.

From W. Storage and J Ganter

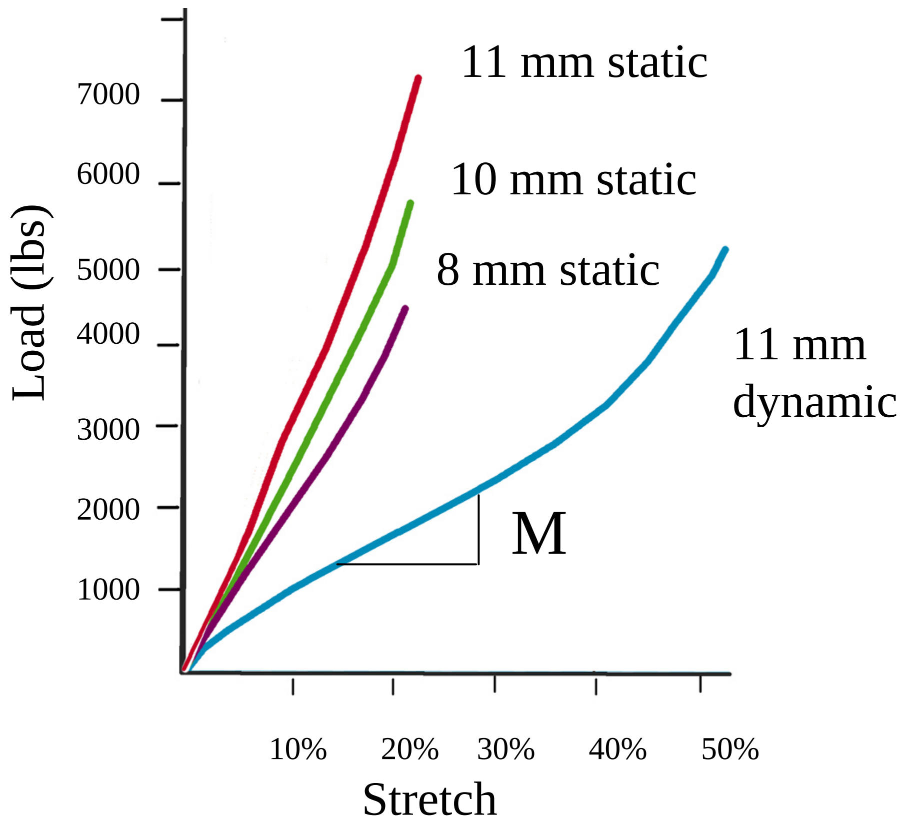

Figure 12: Tensile load for different ropes as a function of rope stretch.(William Strorage and John Ganter)

The strain energy in a rope is energy that is stored as the rope is stretched. A good analogy is the rubber band on a sling-shot. As the rubber is pulled back, the stretching stores elastic energy that is released when the sling shot is released. For the sling shot, the strain energy is converted to kinetic energy and results in the projectile being ‘shot’.

The strain energy is the energy that is stored as rope is stretched. The energy is the product of the stretching force times the distance that the rope elongates.

$$SE = \int \tau d\delta \tag{19}$$

Substituting equation 14 and 15 in to 19 gives

$$SE = \int \frac{\tau L}{M} d\tau \text{ or} \tag{20}$$

$$SE = \frac{L}{2M} T^2 \tag{21}$$

Equation 21 allows the strain energy to be calculated once the load and rope type is known. From these formula, we see that the longer the rope, the more strain energy stored for a given tension. A stiffer rope will have a higher modulus and will not store as much strain energy. (You do not see sling-shots made of steel.) The amount of strain energy depends on the square of the tension.

Recall from figure 1 that the static coefficient of friction (non-sliding) is higher than the dynamic coefficient of friction (sliding). When the sliding starts, the litter will jerk upwards, even if the haul team is pulling smoothly.

When the static slipping force is exceeded, the energy is released. The strain energy is exchanged for kinetic energy. This results in a rescue load being sling-shot upwards. The change in strain energy can be computed by solving for the tension in the ropes before and after the slip.

$$\Delta SE = \frac{L(T_b^2 - T_a^2)}{2M} \tag{22}$$

where T_b and T_a are the rope tensions before and after slip. If we assume that the change in strain energy is converted in potential energy (bounce), then the height of bounce is given by

$$\Delta SE = \Delta PE = Wh \tag{23}$$

where W is the weigh and h is the change in height.

Figure 13 shows a typical three-point contact problem for a lip. Shown in Table 1 is an Excel spread sheet that can be used to compute the tension in the rope, as well as the bounce that results when the rope slips.

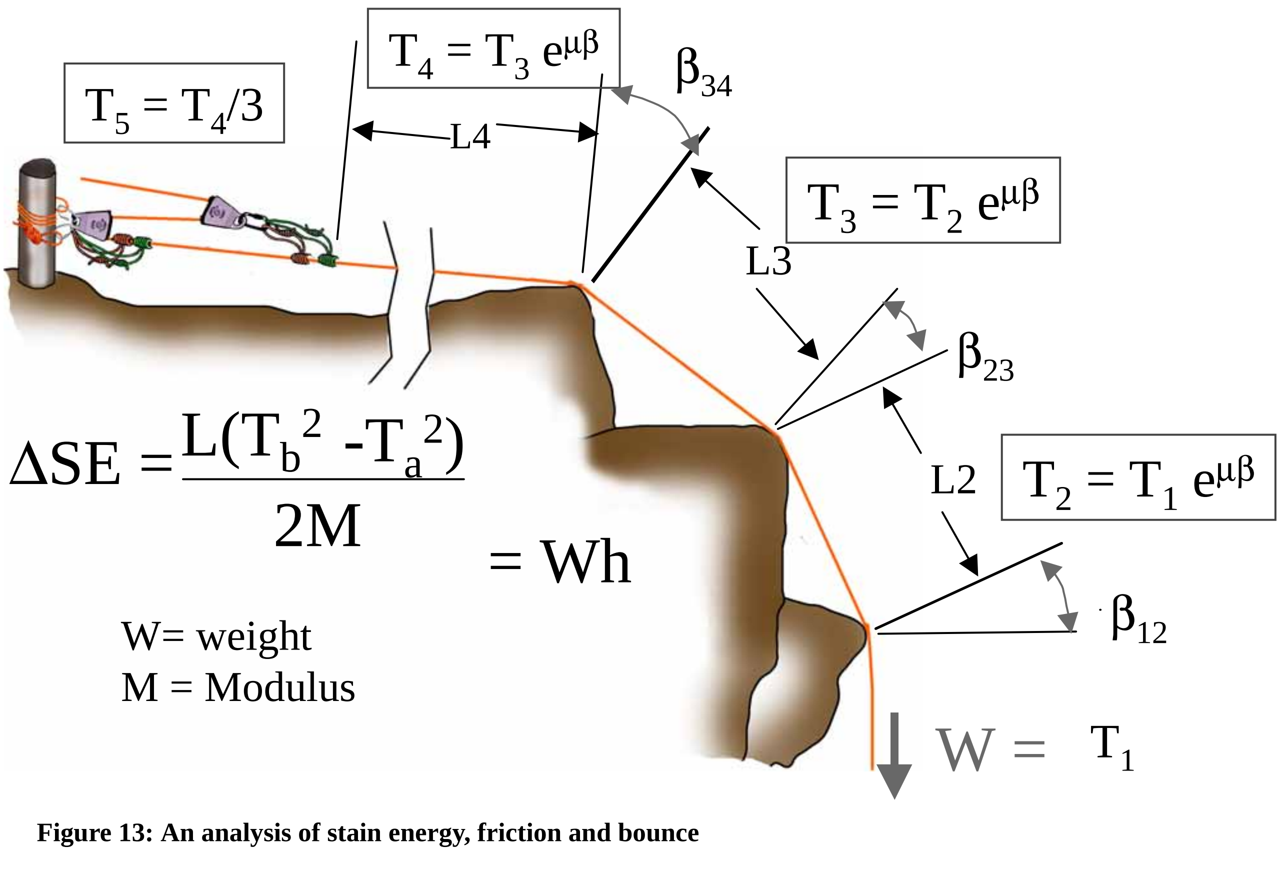

Figure 13: An analysis of stain energy, friction and bounce

Different lengths, angles, and coefficients can be entered, and the table will re-compute the forces and the bounce.

The change in strain energy before and after the slip is assumed to be equal to the change in the potential energy. The bounce will be roughly twice the height of the change in potential energy predicted.

Table 1: Tension and Bounce for Z-System.

Rope Segment	Length (feet)	Change in angle (degree	Static Coefficient of frictio	Dynamic Coefficient of fric	Tension Before Slip	Tension after slip	Modulus (lbs/ft/f	Change in strain energy	bounce (feet) = 2*(
T1	0				600	600		0	
T2	100	30	0.35	0.25	721	684	19555	132.03	
T3	100	30	0.35	0.25	866	780	19555	362.022	
T4	150	30	0.35	0.25	1040	889	19555	1117.75	
Z System	80				347	296	19555	66.237	
Total								1678.04	5.59

When the haul system is tensioned, the rope will slide at the top two points. However, the lowest point will not slip until the friction is exceeded. The strain will build up, and energy will be stored in the rope. When the tension in the rope exceeds the slip tension, the rescue load will slip. This motion reduces the coefficient of friction and allows the rope to slide past the friction points until the tension is reduced. This sudden slip will occur even if the haul rate is very slow.

This stick/slip type motion is similar to the dynamic release of energy during an earthquake. The rope stores its energy until the slip friction is exceeded and allows it to release. For 600-foot haul systems, a bounce of over 6 feet is not uncommon.

Since the dynamic bounce due to stick/slip depends on the square of the tension, the best way to minimize this bounce is to reduce the tension in the system. Using fewer litter attendants, lighter liters, and hauling packs separately will all reduce the primary load. Reducing the friction by use of pulleys and edge rollers will also reduce the total tension in the system. Dynamic bounce can also be reduced by using shorter hauls. A short Z-system may have to be reset more often; however, short Z's will not store as much strain energy. In cases where system dynamics cannot be avoided, the litter attendants should be aware that they will experience a bouncy ride.

Summary

Friction force in a rope depends primarily on three things:

- The load on the rope
- The coefficient of friction
- The angle that the rope turns through

The angle and coefficient of friction can cause an exponential change in rope tension!

We have used a simple friction law for a flexible belt over a drum and applied it to several different rescue situations. The exponential change in rope tension that results from a change in contact angle or coefficient of friction can drastically affect the behavior of all rescue systems.

As a first example, we compared the ideal frictional force for lowering devices. The break tube, the rappel rack, and figure-eight all can work well when supporting loads in the 100- to 200-pound range, provided that 10 to 20 pounds of breaking force can be maintained. However, when much higher rescue loads are involved, we see that the tube has the best ratio of breaking force to rescue load. The adjustable spacing on the bars of the rack give it a great range of frictional force.

The frictional forces that are generated as a rope goes over a lip were computed using the capstan friction equation. While it is counter intuitive, the capstan frictional equation predicts that force going over a edge is independent of the edge radius. The validity of these equations still need to be field tested to see if these ideal friction laws indeed apply to rescue rope.

The exponential change in friction forces with the angle of bend is quite surprising and can catch the most experienced rescuer off guard.

1. *Engineering Mechanics Volume 1, STATCS*, J. L. Meriam, J. Wiley & Sons, pp 301-302, 1978.
2. *Physics for Cavers: Rope, Loads and Energy*, William Strorage and John Ganter, Http://nerve-net.zocalo.net/jg/c/pubs/Rlenergy/Ropesloads.htm